Test 1 - MTH 1310

Dr. Graham-Squire, Summer 2012

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14

Name: Key

ID Number: _____

I pledge that I have neither given nor received any unauthorized assistance on this exam.

_____

(signature)

DIRECTIONS

1. Show all of your work and use correct notation. A correct answer with insufficient work or incorrect notation will lose points.
2. Clearly indicate your answer by putting a box around it.
3. Cell phones and computers are <u>not</u> allowed on this test. Calculators <u>are</u> allowed on all parts of the test, however you should still show all of your work.
4. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
5. If you need to use the quadratic formula, it is $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
6. Make sure you sign the pledge and write your ID on both pages.
7. Number of questions = 7. Total Points = 70.

ID Number: Key Test 1

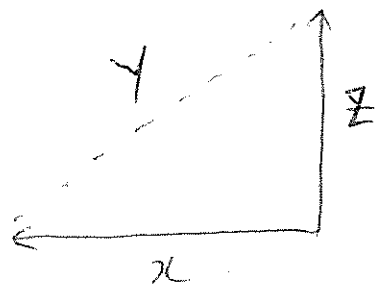
1. (10 points) Two Zaks leave a house at the same time, one goes straight north and the other goes straight west. The north-going Zak travels at 9 miles per hour and the west-going Zak travels at 12 miles per hour.

   (a) Draw a diagram to represent the situation, and let y be the distance between the two Zaks.

   (b) Find an expression for y in terms of t, where t is time (in hours), and $t = 0$ corresponds to when the Zaks left the house.

   (c) Find the distance between the Zaks after 2 hours.

(a)



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(b) $x = 12t$, $z = 9t$ $\Rightarrow y^2 = (12t)^2 + (9t)^2$ ✓ ✓ ✓ ✓

$y = \sqrt{144t^2 + 81t^2}$

$= \sqrt{225t^2}$ ✓

$\boxed{y = 15t}$

5

(c) $y = 15(2) = \boxed{30 \text{ miles}}$

2

2. (10 points) The prison population in the US has grown sharply in the past 40 years. In the 1970s, the number of inmates in prison began to exceed the capacity that the prisons were built for. The number of prisoners (in thousands) in US prisons is approximated by the function

$$N(t) = 3.5t^2 + 26.7t + 436.2 \qquad (0 \le t \le 10)$$

where t is measured in years, with $t = 0$ corresponding to 1983. The number of inmates for which the prisons were designed is given by

$$C(t) = 24.3t + 365 \qquad (0 \le t \le 10)$$

where $C(t)$ is also measured in thousands.

(a) Find an expression that shows the gap between the number of prisoners and the number of inmates the prisons were built for. Simplify your expression by combining like terms.

(b) Find the gap in 1983 and in 1990.

$G_1(t) = N(t) - C(t)$

(a) $N(t) - C(t) = 3.5t^2 + 26.7t + 436.2 - (24.3t + 365)$ ✓

$G(t) = 3.5t^2 + 2.4t + 71.2$

(b) 1983 is $t = 0 \Rightarrow G(0) = 71.2$ thousand $\Rightarrow$ 71,200 ✓✓

@ 1990 is $t = 7 \Rightarrow G(7) = 259.5$

$\Rightarrow$ 259,500 ✓✓

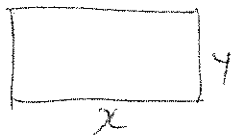
as the perimeter of the area of coop

3. (10 points) Aya wants to build a rectangular fence for her backyard chickens. Since she has 8 chickens, she needs to have an area of 200 ft^2 enclosed by the fence. Let x denote the length of the chicken fence and y denote the width.

   (a) Draw a diagram of the situation, and write an equation relating x and y.

   (b) Find a function $f(x)$ giving the total amount of fencing needed to build the rectangle in terms of only the variable x.

   (c) What is the domain of the function f?

(a) 

$xy = 200 \Rightarrow y = \frac{200}{x}$ ✓✓✓

(b) total amount of fence $= 2x + 2y$

$= 2x + 2\left(\frac{200}{x}\right)$ ✓✓✓

$T(x) = \boxed{2x + \frac{400}{x}}$

(c) domain is $\boxed{x > 0}$ b/c can't divide by zero

✓✓

Be sure to use correct notation.

4. (8 points) Calculate the limits. If the limit does not exist, write DNE and explain your reasoning.

(a) $\lim_{x\to(-\infty)} \dfrac{5x^3 - 8x^2}{7x^3 + 3x^2 + 2x}$ $\cdot \frac{1}{x^3}$ $\cdot \frac{1}{x^3}$

$= \lim_{x\to(-\infty)} \dfrac{5 - \frac{8}{x}}{7 + \frac{3}{x} + \frac{2}{x^2}} = \boxed{\dfrac{5}{7}}$

(b) $\lim_{x\to\infty} \dfrac{3x^7 + 4x - 8}{7x^3 + x}$ $\cdot \frac{1}{x^3}$ $\cdot \frac{1}{x^3}$

$= \lim_{x\to\infty} \dfrac{3x^4 + \frac{4}{x^2} - \frac{8}{x^3}}{7 + \frac{1}{x^2}}$

$= \dfrac{3(\infty)^4 + 0 - 0}{7 + 0}$

does not exist b/c top goes to infinity.

5. (12 points) Calculate the limits. If the limit does not exist, write DNE and explain your reasoning.

(a) $\lim_{x\to 3^-} \frac{t^2-9}{t^3} = \frac{3^2-9}{27} = \frac{0}{27} = 0$

(b) $\lim_{x\to 5} \frac{x^2-5x}{x^2-10x+25} = \lim_{x\to 5} \frac{x(x-5)}{(x-5)(x-5)} = \frac{5}{0}$

$= \lim_{x\to 5} \frac{x}{x-5}$

So limit DNE because it is a # divided by zero.

(c) $\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4}$. Hint: rationalize the numerator. $\frac{2-2}{4-4} = \frac{0}{0}$

$\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2}$

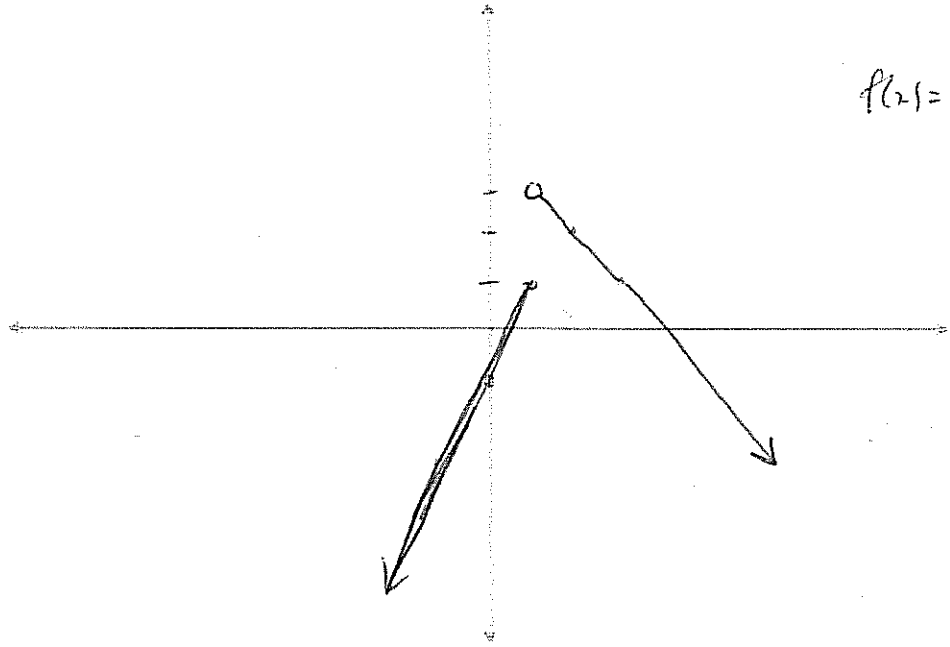
$= \lim_{x\to 4} \frac{x-4}{(x-4)(\sqrt{x}+2)}$

$= \lim_{x\to 4} \frac{1}{\sqrt{x}+2} = \frac{1}{2+2} = \frac{1}{4}$

6. (15 points) Sketch a graph of the following function $f(x)$, then calculate the limits below. If the limit does not exist, write DNE and explain your reasoning.

Need graph function

$$f(x) = \begin{cases} 2x-1 & \text{if } x \leq 1 \\ 4-x & \text{if } x > 1 \end{cases}$$



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(a) $\lim_{x \to 1^+} f(x) =$ 3

2

(b) $\lim_{x \to 1^-} f(x) =$ 1

2

(c) $\lim_{x \to 1} f(x) =$ DNE b/c the one-sided limits are not equal.

2

3 (d) Continuity

7. (5 points) Find an equation for the line connecting the points (2,7) and (-1, 5). Write your answer in the form $y =$ _____.

$$m = \frac{5-7}{-1-2} = \frac{-2}{-3}$$

$$y - 7 = \frac{2}{3}(x-2)$$

$$y = \frac{2}{3}x - \frac{4}{3} + 7$$

$$y = \frac{2}{3}x + \frac{17}{3}$$

Extra Credit(2 points) Let $h(x) = \dfrac{(x^2+1)^3+4}{x^2}$. Find functions $f(x)$ and $g(x)$ so that $h(x) = f(g(x))$.

$g(x) = x^2 + 1$ and $f(x) = \dfrac{x^3+4}{x-1}$

$g(x) = x^2$ and $f(x) = \dfrac{(x+1)^3+4}{x}$